

choose then pick
stochastic choice with admissibility and
residual selection

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choice correspondences are sets

choice correspondences are set-valued, e.g.:

$$C(\{A, B, C\}) = \{A, B\}$$

typically cannot literally pick $\{A, B\}$

picks A or B , but which one?

canonically, dm indifferent between A and B ,
so selection doesn't affect dm's welfare...

however, this may not be the case if the dm is
constrained in their ability to discriminate

choose, then pick

this paper models a single choice as two stages:

- ▶ **choose** – which alternatives are **admissible**;
- ▶ **pick** – select one of them.

the choosing/picking distinction is from the philosophy of action (Ullmann-Margalit and Morgenbesser, 1977).

picking vs. choosing

we focus on:

- ▶ **semiorders** for the choose stage; captures imperfect evaluation;
- ▶ (broadly) **stochastic choice** for selection among options

these are natural models that typically do not interact in the literature (with some exceptions)

semiorders

- ▶ Luce (1956): discrimination has a threshold δ (a “just-noticeable difference”)
- ▶ x is ranked above y only if $u(x) > u(y) + \delta$; closer alternatives are left **unresolved**.



why semiorders?

- ▶ reflects incompleteness stemming from imperfect discrimination
- ▶ natural case where multi-valued choice correspondences, i.e. the set of **admissible alternatives**, arises

why semiorders?

- ▶ reflects incompleteness stemming from imperfect discrimination
- ▶ natural case where multi-valued choice correspondences, i.e. the set of **admissible alternatives**, arises

...but **silent** on

1. ranking within admissible alternatives
2. which alternative is finally selected

solutions

1. get rankings within admissible alternatives using nested semiorders (Betto, 2026)
2. get final selections with stochastic choice (Luce, 1959; McFadden, 1974)

revealed preference results

choose stage

- ▶ under full support (PA), preferences identified and admissible iff
 1. no-crossing supports
 2. revealed-best coherence
- ▶ without PA, can only **bound** admissibility

pick stage

- ▶ **cyclical independence** of within-core odds
⇒ luce weights identified

revealed preference results

$\{p_\lambda\}$ is **choose-then-pick with luce picking** iff
(1) no-crossing supports, (2) revealed-best
coherence, and (3) cyclical independence

sharper discrimination **improves** choices: λ'
more discriminating than $\lambda \Rightarrow p_{\lambda'}(\cdot, S) \text{ FOSD } p_\lambda(\cdot, S)$
in the benchmark order.

holds **whatever** the pick weights – the welfare lever
behind defaults and order sets.

Related literature

choose then pick

- ▶ choose/pick with consistent preferences (Horan, 2021)

pick then choose

- ▶ Manzini and Mariotti (2007, 2014); Masatlioglu et al. (2012); Lleras et al. (2017); Cattaneo et al. (2020); Cherepanov et al. (2013); Kalai et al. (2002)

semiorders / coarse discrimination

- ▶ Betto (2026); Luce (1956); Scott and Suppes (1958); Fishburn (1970); Roberts (1971)

model

objects

- ▶ finite alternatives: X .
- ▶ nonempty menus: $\mathcal{S} = 2^X \setminus \{\emptyset\}$.
- ▶ environments: $\lambda \in \Lambda$.

interpretation of λ

λ indexes **discriminability**: how finely benchmark differences are resolved in a given environment.

“division of labor” between choosing and picking: **discriminability** parameter, λ

- ▶ higher discriminability \Rightarrow fewer survivors
 \Rightarrow **less picking**
- ▶ lower discriminability \Rightarrow more survivors
 \Rightarrow **more picking**

definition: discriminability-dependent admissibility

A family $C = \{C_\lambda : \lambda \in \Lambda\}$ admits a **discriminability-dependent representation** if there exist $u: X \rightarrow \mathbb{R}$, \succeq on Λ , and $\kappa_\lambda: u(X) \rightarrow \mathbb{R}$ such that

$$C_\lambda(S) = \{x \in S : u(x) \geq \kappa_\lambda(M_u(S))\}, \quad M_u(S) := \max_{z \in S} u(z),$$

and

- (i) $\kappa_{\bar{\lambda}}(m) = m$,
- (ii) $\lambda' \succeq \lambda \Rightarrow \kappa_{\lambda'}(m) \geq \kappa_\lambda(m)$,
- (iii) $m' \geq m \Rightarrow \kappa_\lambda(m') \geq \kappa_\lambda(m)$.

- ▶ u is an ordinal **benchmark ranking**.
- ▶ $M_u(S)$ is the utility of the best alternative in the menu.
- ▶ $\kappa_\lambda(M_u(S))$ is a cutoff induced by that best alternative.
- ▶ more discriminating environments have higher cutoffs.
- ▶ at $\bar{\lambda}$, admissibility is exact maximization:

$$C_{\bar{\lambda}}(S) = \operatorname{argmax}_{x \in S} u(x).$$

two immediate implications

top segments

if $x \in C_\lambda(S)$ and $u(y) \geq u(x)$ for $y \in S$, then $y \in C_\lambda(S)$.

contraction (ia)

if $T \subseteq S$, then

$$C_\lambda(S) \cap T \subseteq C_\lambda(T).$$

output of the choose stage

the output of this stage is the admissible set

$$C_\lambda(S).$$

$C_\lambda(S)$ records reason-guided evaluation.

- ▶ singleton \Rightarrow decision fully resolved by reasons.
- ▶ non-singleton \Rightarrow first-order reasons leave ties.

pick stage

a second-stage rule selects one alternative from $C_\lambda(S)$:

- ▶ e.g. default rule, positional convention, coin flip, randomization, ...
- ▶ captures procedural forces but not preferences.

the output of this stage is a selection probability.

definition: pick rule

fix an environment λ and an admissibility correspondence C_λ

a map

$$p_\lambda : X \times \mathcal{S} \rightarrow [0, 1]$$

is a pick rule on C_λ if, for every menu $S \in \mathcal{S}$,

$$p_\lambda(x, S) = 0 \quad \text{for all } x \notin C_\lambda(S),$$

and

$$\sum_{x \in C_\lambda(S)} p_\lambda(x, S) = 1.$$

example 1

default

$$\delta(\mathcal{S}) \in \mathcal{S},$$

parameter

$$\alpha \in (0, 1).$$

$$p_\lambda(\mathbf{x}, \mathcal{S}) = \begin{cases} 1 - \alpha + \frac{\alpha}{|\mathcal{C}_\lambda(\mathcal{S})|}, & \mathbf{x} = \delta(\mathcal{S}) \in \mathcal{C}_\lambda(\mathcal{S}), \\ \frac{\alpha}{|\mathcal{C}_\lambda(\mathcal{S})|}, & \mathbf{x} \in \mathcal{C}_\lambda(\mathcal{S}) \setminus \{\delta(\mathcal{S})\}, \quad \delta(\mathcal{S}) \in \mathcal{C}_\lambda(\mathcal{S}), \\ \frac{1}{|\mathcal{C}_\lambda(\mathcal{S})|}, & \mathbf{x} \in \mathcal{C}_\lambda(\mathcal{S}), \quad \delta(\mathcal{S}) \notin \mathcal{C}_\lambda(\mathcal{S}), \\ 0, & \mathbf{x} \notin \mathcal{C}_\lambda(\mathcal{S}). \end{cases}$$

example 2

fix exogenous total order \triangleright on X , e.g. alphabetical, screen position.

$$p_\lambda(x, S) = \begin{cases} 1, & x = \max_{\triangleright} C_\lambda(S), \\ 0, & \text{otherwise.} \end{cases}$$

- ▶ only one admissible alternative gets positive probability.
- ▶ selected because of preferences or because of position?

definition: choose-then-pick representation

a stochastic choice profile

$$\mathcal{P} = \{p_\lambda : \lambda \in \Lambda\}$$

has a choose-then-pick representation if there is a family

$$\mathcal{C} = \{C_\lambda : \lambda \in \Lambda\}$$

such that:

- (i) \mathcal{C} admits a discriminability-dependent representation;
- (ii) for every $\lambda \in \Lambda$, p_λ is a pick rule on C_λ .

representation separates:

benchmark ranking u

admissible sets $C_\lambda(S)$

pick rule $p_\lambda(\cdot, S)$

The pick rule can be uniform, Luce, default-biased, priority-based, or deterministic.

Observable support:

$$D_\lambda(S) := \text{supp}_\lambda(S) = \{x \in S : p_\lambda(x, S) > 0\}.$$

By admissibility of picking,

$$D_\lambda(S) \subseteq C_\lambda(S).$$

positivity of admissibles (PA)

$$C_\lambda(S) = D_\lambda(S) \quad \text{for all } \lambda, S.$$

- ▶ PA is full support on the admissible set
- ▶ if an alternative is admissible, it is picked with positive probability.

satisfies PA

uniform picking; Luce
with positive weights;
stochastic priority
with positive rank
weights.

may violate PA

deterministic
priority;
deterministic default;
degenerate picking.

a pick rule on survivors

given the admissible set, a **pick** rule selects one of its elements:

$$\sum_{x \in C_\lambda(S)} p_\lambda(x, S) = 1, \quad p_\lambda(x, S) = 0 \quad \text{for } x \notin C_\lambda(S).$$

nothing yet restricts **how** it picks – a default, a position, a priority order, a coin flip...

today: luce picking

the pick stage can be anything supported on the core.

today we focus on one class: luce.

it is the natural stochastic benchmark, it nests many cases we care about, and – as we will see – it is exactly identified from within-core odds

luce weights (luce 1959)

give each alternative a positive **weight** and choose in proportion to weights:

$$p(x, S) = \frac{w(x)}{\sum_{z \in S} w(z)}.$$

equivalently, the **odds** between two alternatives are constant:

$$\frac{p(x, S)}{p(y, S)} = \frac{w(x)}{w(y)}.$$

luce *inside* the core

we apply luce **only among the survivors**: pick weights $\tau : X \rightarrow \mathbb{R}_{++}$ with

$$p_\lambda(x, S) = \frac{\tau(x)}{\sum_{z \in C_\lambda(S)} \tau(z)} \quad \text{for } x \in C_\lambda(S), \quad 0 \text{ otherwise.}$$

- ▶ odds are luce **within** the core; the core itself moves with λ ;
- ▶ crucially, τ **need not agree with** u – picking is procedure, not preference.

revealed preference results

the choose stage

the choose stage is **semiorder-based**: coarse ranking fixes the admissible set

partial optimization: deterministic given the menu, the ranking, and discriminability – so all stochasticity sits in the pick stage

If a choose-then-pick representation satisfies PA, then

$$C_\lambda(S) = D_\lambda(S) \quad \text{for every } \lambda, S.$$

- ▶ without PA: support is only a lower bound.
- ▶ with PA: support is the admissible set.
- ▶ the choose-stage question becomes observable: when can $D = \{D_\lambda\}$ be represented by benchmark cutoffs?

axiom: no-crossing supports

If some alternative is supported under λ but not under λ' ,

$$p_\lambda(x, S) > 0 \neq p_{\lambda'}(x, S),$$

then every support under λ' is contained in the corresponding support under λ :

$$p_{\lambda'}(y, T) > 0 \Rightarrow p_\lambda(y, T) > 0 \quad \text{for all } y, T.$$

- ▶ λ' excludes something that λ admits
- ▶ then λ' must be weakly more discriminating *everywhere*
- ▶ the axiom rules out crossing support patterns
- ▶ comes from treating λ as a one-dimensional discriminability index

no-crossing supports gives an order over environments:

$$\lambda' \preceq^D \lambda \iff D_{\lambda'}(S) \subseteq D_{\lambda}(S) \text{ for every } S.$$

λ' is revealed to be at least as discriminating as λ .

under no-crossing, \preceq^D is a weak order on support-equivalence classes.

axiom: revealed-best coherence

take $x, y \in S \cap T$. if

$$p_{\mu}(x, S) > 0 \quad \text{for every } \mu \in \Lambda$$

and

$$p_{\lambda}(y, T) > 0,$$

then coherence requires

$$p_{\lambda}(x, T) > 0 \quad \text{and} \quad p_{\lambda}(y, S) > 0.$$

- ▶ x is revealed-best in S if it is chosen with positive probability in every environment:

$$p_{\mu}(x, S) > 0 \quad \text{for every } \mu \in \Lambda.$$

- ▶ if x is revealed-best in S , and y is chosen from another menu T with positive probability in λ , then:
 - x must also be chosen wpp from T ;
 - y must also be chosen wpp from S .

theorem

under PA, the choose stage is identified from support and admits a discriminability-dependent cutoff representation iff:

- (i) no-crossing supports holds;
- (ii) revealed-best coherence holds.

identified choose-stage objects

when the theorem applies, support identifies:

- ▶ admissible sets $C_\lambda(S) = D_\lambda(S)$;
- ▶ revealed discriminability order \succeq^D ;
- ▶ benchmark weak order from a maximally discriminating class:

$$x \succeq^D y \iff x \in D_{\bar{\lambda}}(\{x, y\}).$$

the pick stage

the pick rule selects **one admissible** alternative – and it can be almost anything: a default, screen position, alphabetic order, salience, familiarity, randomization...

the pick stage

the pick rule selects **one admissible** alternative – and it can be almost anything: a default, screen position, alphabetic order, salience, familiarity, randomization...

but **only admissible alternatives are ever picked**

inside the recovered core

Under PA and the support theorem,

$$C_\lambda(S) = D_\lambda(S).$$

Now the remaining question is about probabilities inside $C_\lambda(S)$.

For $x, y \in C_\lambda(S)$, define odds:

$$R_\lambda(x, y; S) = \frac{p_\lambda(x, S)}{p_\lambda(y, S)}.$$

definition: co-admissibility graph

For each λ , define $G_\lambda = (X, E_\lambda)$ by

$$\{x, y\} \in E_\lambda \iff x \neq y \text{ and } \exists S \text{ with } x, y \in C_\lambda(S).$$

The global graph is

$$G = (X, E), \quad E = \bigcup_{\lambda \in \Lambda} E_\lambda.$$

- ▶ an edge means the pair is ever jointly admissible.
- ▶ only co-admissible pairs generate observable within-core odds.
- ▶ if G is disconnected, pick-weight ratios are not identified across components.

no co-admissibility \Rightarrow no odds comparison.

axiom: cyclical independence at λ

For any cycle of co-admissible odds at fixed λ ,

$$\{(x_i, S_i)\}_{i=1}^n, \quad x_i, x_{i+1} \in C_\lambda(S_i), \quad x_{n+1} = x_1,$$

require

$$\prod_{i=1}^n \frac{p_\lambda(x_i, S_i)}{p_\lambda(x_{i+1}, S_i)} = 1.$$

- ▶ local odds must be path independent.
- ▶ if x and y are co-admissible in two menus, their odds agree.
- ▶ longer cycles impose consistency across overlapping comparisons.

[generalized Luce cycle condition of Ahumada-Ülkü and Echenique-Saito, applied to the recovered core]

Proposition (Ahumada-Ülkü; Echenique-Saito)

For fixed λ and known C_λ , cyclical independence holds iff there is $\tau_\lambda : X \rightarrow \mathbb{R}_{++}$ such that

$$p_\lambda(x, S) = \frac{\tau_\lambda(x)}{\sum_{z \in C_\lambda(S)} \tau_\lambda(z)} \quad \text{for } x \in C_\lambda(S).$$

τ_λ is unique up to scale on each component of G_λ .

proof

- ▶ generalized Luce tells us when odds come from weights.
- ▶ the choose-then-pick model tells us what the core means.
- ▶ $C_\lambda(S)$ is not arbitrary: it is the set that survived benchmark evaluation.
- ▶ τ_λ is a pick-stage object, not necessarily the benchmark ranking.

axiom: global cyclical independence

Now allow the cycle to move across environments:

$$\{(\mathbf{x}_i, \mathcal{S}_i, \lambda_i)\}_{i=1}^n, \quad \mathbf{x}_i, \mathbf{x}_{i+1} \in \mathcal{C}_{\lambda_i}(\mathcal{S}_i).$$

Require

$$\prod_{i=1}^n \frac{p_{\lambda_i}(\mathbf{x}_i, \mathcal{S}_i)}{p_{\lambda_i}(\mathbf{x}_{i+1}, \mathcal{S}_i)} = 1.$$

- ▶ all local odds can be rationalized by one common system of weights.
- ▶ if λ changes admissibility but not picking, odds among survivors stay stable.
- ▶ if the procedure also changes, global CI may fail.

result: common within-core Luce

theorem

take $\mathcal{C} = \{C_\lambda\}$ as fixed, with full support on each core. global cyclical independence holds iff there is one $\tau: X \rightarrow \mathbb{R}_{++}$ such that

$$p_\lambda(x, S) = \frac{\tau(x)}{\sum_{z \in C_\lambda(S)} \tau(z)} \quad \text{for } x \in C_\lambda(S).$$

τ is unique up to scale on each connected component of G .

proof

axiom: core renormalization

If

$$C_{\lambda'}(S) \subseteq C_{\lambda}(S),$$

then probabilities at λ' are obtained by conditioning on the smaller core:

$$p_{\lambda'}(\mathbf{x}, S) = \frac{p_{\lambda}(\mathbf{x}, S)}{p_{\lambda}(C_{\lambda'}(S), S)} \quad \text{for } \mathbf{x} \in C_{\lambda'}(S).$$

- ▶ increasing discriminability deletes alternatives from the core.
- ▶ it does not reweight alternatives that remain.
- ▶ if the same core remains, probabilities remain the same.

Corollary

If admissible cores satisfy no crossing, then the following are equivalent:

- (i) one common within-core Luce representation;
- (ii) fixed- λ cyclical independence plus core renormalization;
- (iii) global cyclical independence.

proof

under common τ uce, odds identify τ .

But nothing requires

$$u(x) \geq u(y) \iff \tau(x) \geq \tau(y).$$

- ▶ u : benchmark evaluation.
- ▶ τ : salience, ease, default status, priority, familiarity.

non-alignment in the example

benchmark order revealed from support:

$$A \succ^D B \succ^D C.$$

observed odds:

$$\frac{p(B, \{A, B\})}{p(A, \{A, B\})} = 4, \quad \frac{p(B, \{B, C\})}{p(C, \{B, C\})} = 2.$$

normalizing $\tau(A) = 1$:

$$\tau(A) = 1, \quad \tau(B) = 4, \quad \tau(C) = 2.$$

$$B \succ_{\tau} C \succ_{\tau} A \quad \text{while} \quad A \succ^D B \succ^D C.$$

axiom: support-to-odds monotonicity

If support reveals that z_k is strictly above z_0 ,

$$p_\lambda(z_k, S) > 0 = p_\lambda(z_0, S),$$

then every path of within-core odds from z_0 to z_k must point upward:

$$\prod_{j=0}^{k-1} \frac{p_{\lambda_j}(z_{j+1}, S_j)}{p_{\lambda_j}(z_j, S_j)} > 1.$$

- ▶ support eliminations reveal strict benchmark comparisons.
- ▶ if z_k survives while z_0 does not, then z_k should have larger pick weight in the aligned case.
- ▶ the axiom states this without first assuming Luce weights exist.

axiom: odds-to-support monotonicity

If x is ever strictly more likely than y ,

$$p_{\lambda}(x, S) > p_{\lambda}(y, S),$$

then there is some environment and menu where x survives and y does not:

$$p_{\lambda'}(x, S') > 0 = p_{\lambda'}(y, S'), \quad x, y \in S'.$$

- ▶ strict odds advantages must be visible somewhere in support.
- ▶ this is a richness condition.
- ▶ otherwise odds could rank alternatives more finely than the support data ever do.

theorem

Assume PA, No-Crossing Supports, Revealed-Best Coherence, and connected G . Then the following are equivalent:

1. Global CI, Support-to-Odds Monotonicity, and Odds-to-Support Monotonicity;
2. there exists $v : X \rightarrow \mathbb{R}_{++}$ such that

$$p_\lambda(x, S) = \frac{v(x)}{\sum_{z \in C_\lambda(S)} v(z)}$$

and v represents the benchmark order \succeq^D .

The baseline model separates:

benchmark ranking and pick weights.

Alignment imposes that one index does both jobs.

- ▶ close to threshold Luce and stochastic semiororders;
- ▶ not appropriate when defaults, ease, salience, or priority affect final selection.

extensions and wrap-up

- ▶ what can be learned when PA fails; Roberts completions;
- ▶ deterministic priority and sequential rationales;
- ▶ relation to general Luce, threshold Luce, and stochastic semiorders;
- ▶ proof sketches

no PA extension

related models

proofs

conclusion

- ▶ final choice probabilities can mix benchmark evaluation and procedural selection.
- ▶ under PA, zeros identify admissibility.
- ▶ No-Crossing Supports and Revealed-Best Coherence characterize the choose stage.
- ▶ within-core odds identify picking weights under Luce restrictions.
- ▶ those weights need not agree with the benchmark ranking.

when PA fails

no PA

Without PA,

$$D_\lambda(S) \subseteq C_\lambda(S)$$

but equality need not hold.

Positive probability still reveals admissibility. Zero probability does not reveal inadmissibility.

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definition: admissibility completion

Given observed supports \mathcal{D} , define $\mathfrak{C}(\mathcal{D})$ as all families $\mathcal{C} = \{C_\lambda\}$ such that:

1. \mathcal{C} has a discriminability-dependent cutoff representation;
2. $D_\lambda(S) \subseteq C_\lambda(S)$ for all λ, S .

back

completions

- ▶ observed supports are lower bounds on admissibility.
- ▶ completions add alternatives that may be admissible but unchosen.
- ▶ smaller completions treat more zeros as inadmissibility.
- ▶ larger completions treat more zeros as deterministic or degenerate picking.

result: sharp no-PA identified set

Proposition

If the only pick-stage restriction is admissibility of picking, then

$$\mathcal{C}(\mathcal{D})$$

is the sharp identified set for the admissibility family.

Every completion can rationalize the observed data by using the observed probabilities as the pick rule.

definition: Roberts completion

For each λ , let P_λ be a strict relation.

$$C_\lambda^P(S) = \max_{P_\lambda}(S)$$

where $x \in \max_{P_\lambda}(S)$ iff no feasible y satisfies $yP_\lambda x$.

A Roberts completion is a family $P = \{P_\lambda\}$ that:

- ▶ is support-feasible: $D_\lambda(S) \subseteq \max_{P_\lambda}(S)$;
- ▶ satisfies Roberts-style compatibility: GA, N, UW, SSC.

what Roberts completions do

- ▶ they are choose-stage restrictions, not pick-stage restrictions.
- ▶ they impose one benchmark order and nested discriminability relations.
- ▶ they parameterize the candidate admissibility families consistent with observed support.

result: Roberts parameterization

Theorem

The admissibility families induced by Roberts completions are exactly the sharp no-PA identified set:

$$\{\mathcal{C}^P : P \in \mathcal{R}(\mathcal{D})\} = \mathfrak{C}(\mathcal{D}).$$

[proof](#)

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deterministic priority

A deterministic priority picker selects

$$c_\lambda(S) = \max_{\triangleright} C_\lambda(S).$$

Then observed support is singleton:

$$D_\lambda(S) = \{c_\lambda(S)\}.$$

All lower-priority admissible alternatives are hidden from support.

result: priority as sequential
rationales

Proposition

If

$$C_\lambda(S) = \max_{P_\lambda}(S),$$

and final selection is deterministic priority,
then

$$c_\lambda(S) = \max_{\triangleright}(\max_{P_\lambda}(S)).$$

This is a two-rationale procedure in the sense
of Manzini-Mariotti.

known-priority bounds

If priority \triangleright is known, every rationalizing admissible set satisfies

$$\{c_\lambda(S)\} \subseteq C_\lambda(S) \subseteq \{x \in S : c_\lambda(S) \triangleright x \text{ or } x = c_\lambda(S)\}.$$

[proof](#)

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related models

backup: general Luce and threshold Luce

Echenique-Saito general Luce:

$$p(x, S) = \frac{\tau(x)}{\sum_{z \in c(S)} \tau(z)} \quad \text{for } x \in c(S).$$

In this paper, under PA:

$$c(S) = C_\lambda(S)$$

and the core is interpreted as the set surviving benchmark evaluation.

threshold Luce as aligned cutoff

Threshold Luce survival:

$$(1 + \alpha)\tau(\mathbf{X}) \geq \max_{\mathbf{Z} \in S} \tau(\mathbf{Z}).$$

Taking logs:

$$\log \tau(\mathbf{X}) \geq \max_{\mathbf{Z} \in S} \log \tau(\mathbf{Z}) - \log(1 + \alpha).$$

This is a fixed-environment cutoff rule where the same index determines survival and odds.

Horan: stochastic semiorders

Horan combines:

- ▶ semioorder elimination: remove sufficiently inferior alternatives;
- ▶ Luce probabilities on the survivors.

The aligned fixed- λ version of this model overlaps with that logic.

But the baseline here allows

benchmark ranking $u \neq$ pick weights τ .

two extensions relative to Horan

1. **non-alignment**: the index governing admissibility may differ from the weights governing odds.
2. **cross-environment structure**: admissibility varies with discriminability λ .

A coarse environment may not distinguish a from b , while cross- λ support reveals

$$a \succ^D b \succ^D c.$$

proof sketches

proof: PA identifies admissibility

PA says

$$x \in C_\lambda(S) \iff p_\lambda(x, S) > 0.$$

But the right-hand side is exactly

$$x \in D_\lambda(S).$$

So

$$C_\lambda(S) = D_\lambda(S).$$

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proof: support characterization

- ▶ Necessity: cutoff representation implies supports shrink monotonically with discriminability and satisfy top-segment/cutoff coherence.
- ▶ These are exactly No-Crossing Supports and Revealed-Best Coherence.
- ▶ Sufficiency: apply Betto's CDR representation theorem to the support family D .
- ▶ cutoff form follows from the equivalence between CDR and cutoff representation.

proof: fixed-environment Luce

- ▶ If Luce holds, every odds ratio is $\tau_\lambda(x)/\tau_\lambda(y)$; products around cycles telescope to one.
- ▶ Conversely, fix a root in each component of G_λ .
- ▶ Define $\tau_\lambda(x)$ as the product of observed odds along any path from the root to x .
- ▶ Cyclical independence makes this path-independent.
- ▶ Normalize over each core to get the Luce formula.

proof: common Luce

- ▶ Same path-product argument, but paths now use edges from the global co-admissibility graph G .
- ▶ Global CI guarantees path independence across menus and environments.
- ▶ The constructed τ rationalizes every within-core odds ratio.
- ▶ Normalization on each core yields the common Luce formula.

proof: common weights and renormalization

- ▶ Common Luce immediately gives fixed- λ CI.
- ▶ If a core shrinks, the common weights imply conditioning on the smaller core.
- ▶ Conversely, no crossing gives a least-discriminating environment for any finite cycle.
- ▶ Core renormalization moves all odds in the cycle to that environment.
- ▶ fixed- λ CI then gives global CI, hence common weights.

proof: benchmark-aligned Luce

- ▶ GCI gives common weights τ .
- ▶ Support-to-Odds: strict support comparisons imply $\tau(x) > \tau(y)$.
- ▶ Odds-to-Support: strict τ comparisons must appear as support comparisons.
- ▶ therefore τ represents the benchmark order \preceq^D .
- ▶ conversely, if one positive index v represents \preceq^D and generates Luce odds, all three axioms follow.

proof: sharp no-PA set

- ▶ Any representation must contain all positive-probability alternatives, so $D_\lambda(S) \subseteq C_\lambda(S)$.
- ▶ Therefore every representation lies in $\mathfrak{C}(\mathcal{D})$.
- ▶ Conversely, take any completion $C \in \mathfrak{C}(\mathcal{D})$.
- ▶ Use the observed probabilities themselves as the pick rule.
- ▶ They are supported on C because $D \subseteq C$.

proof: Roberts parameterization

- ▶ A Roberts completion induces $C_{\lambda}^P(S) = \max_{P_{\lambda}}(S)$.
- ▶ Roberts representation gives a discriminability-dependent admissibility representation.
- ▶ support feasibility gives $D \subseteq C^P$.
- ▶ Conversely, any discriminability-dependent completion has an equivalent CDR/tolerance representation.
- ▶ Roberts theorem gives the relation family P that induces it.

proof: priority as sequential rationales

If

$$C_\lambda(S) = \max_{P_\lambda}(S),$$

then deterministic priority gives

$$c_\lambda(S) = \max_{\triangleright} C_\lambda(S) = \max_{\triangleright} (\max_{P_\lambda}(S)).$$

This is exactly the sequential application of the first rationale P_λ and the second rationale \triangleright .

proof: known-priority bounds

- ▶ selected alternative must be admissible:

$$c_\lambda(S) \in C_\lambda(S).$$

- ▶ any admissible alternative with priority above $c_\lambda(S)$ would have been selected instead.
- ▶ therefore it cannot be admissible.
- ▶ any set satisfying these two bounds rationalizes the fixed-menu choice.

picking vs. choosing

Ullmann-Margalit and Morgenbesser (1977) draw the line at **preference**: one **chooses** when the act “is determined by the differences in one’s preferences over them.”

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one **picks** when there is no such difference to go on – the alternatives are symmetric, one is indifferent, yet a single one must still be taken, as with buridan’s ass between two equal bales.

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