Log-Logistic Demand

Stable Estimation Under Technological Progress

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Technological change can dramatically shift costs (and prices)

This may be direct

- E.g., cheaper LCD panels affect all TV manufacturers

It also may be due to a change in the *units of quantity*

- Upcoming batteries predicted to have twice the watt-hours
- New hard drives with HAMR slated to have twice the TB

Motivating example: vertical merger threatens availability of new technology

Central question: how can/should cost changes affect demand calibration

Review: logit and discrete choice

Discrete choice microfoundation

DM chooses alternative with highest utility given by

$$U_i(\mathbf{p}) = u_i(p_i) + \underbrace{\varepsilon_i}_{\text{i.i.d. Gumbel}}$$

The probability that i is selected is

$$\Pr\left\{u_i(p_i) + \varepsilon_i \geq \max_{k \neq i} u_k(p_k) + \varepsilon_k\right\} = \frac{\exp(u_i(p_i))}{\sum_k \exp(u_k(p_k))}$$

Max of Gumbels is Gumbel and difference of Gumbels is Logit

What practitioners do and what they could do

Linear - logit

Usual assumption, $u_i(p_i) \equiv \gamma_i + \beta p_i$

Choice between *i* and *k* determined by price difference

$$u_i(p_i) - u_k(p_k) = \gamma_i - \gamma_k + \beta(p_i - p_k)$$

Log - logit

We could assume, $u_i(p_i) \equiv \log(\delta_i) + \beta \log(p_i)$

Then choice between *i* and *k* determined by price *ratio*

$$u_i(p_i) - u_k(p_k) = \log\left[\frac{\delta_i}{\delta_k}\left(\frac{p_i}{p_k}\right)^{\beta}\right]$$

Standard logistic demand

Logit sensitive to absolute price effects (first-order intuition)

Take some logit demand for storage with prices in $/TB

$$s_i(\mathbf{p}) \equiv \frac{\exp(\gamma_i - \beta p_i)}{\sum_k \exp(\gamma_k - \beta p_k)}$$

If innovation doubles the capacity of each drive, price per TB is half:

$$\begin{aligned} s_i(\mathbf{p}/2) &= \frac{\exp(\gamma_i - \beta p_i/2)}{\sum_k \exp(\gamma_k - \beta p_k/2)} \\ &= s_i(\mathbf{p}) \exp(\beta p_i/2) \frac{\sum_k \exp(\gamma_k - \beta p_k)}{\sum_k \exp(\gamma_k - \beta p_k/2)} \\ &= \frac{s_i(\mathbf{p}) \exp(\beta p_i/2)}{\sum_k s_k(\mathbf{p}) \exp(\beta p_k/2)} \end{aligned}$$

Halving prices increases shares of more expensive products

Concrete example with price adjustment

Prices are endogenous

Firm	Demand	MC	Price	Share
1	$\frac{\exp(1-p_1)}{\exp(1-p_1)+\exp(2-p_2)}$	1	3	50%
2	$\frac{\exp(2-p_2)}{\exp(1-p_1)+\exp(2-p_2)}$	2	4	50%

Reducing marginal cost (usually) increases share of higher priced item

Firm	Demand	MC	Price	Share
1	$\frac{\exp(1-p_1)}{\exp(1-p_1)+\exp(2-p_2)}$	0.5	2.35	46%
2	$\frac{\exp(2-p_2)}{\exp(1-p_1)+\exp(2-p_2)}$	1	3.18	54%

Logit sensitive to choice of units

If demand is measured in $/hardrive instead of $/TB,

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Choice of quantity units (storage vs drives) changes effect of innovation on market.

Is this reasonable?

You could adjust β, but this is not a solution

Common "solution" is to double beta when you cut costs in half,

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but this assumes "true" demand is in hard drives instead of storage.

Firm	Demand	MC	Price	Share
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Is that better? Why hack around the quirks of logit, when we could fix them?!

Solution: Log-logistic demand

Log-logistic demand isn't so different from logistic demand

Logistic demand looks like

$$s_i(\mathbf{p}) \equiv \frac{\exp(\gamma_i - \beta p_i)}{\sum_k \exp(\gamma_k - \beta p_k)}$$

Log-logistic demand isn't so different from logistic demand

But I'm going to write it like this for the rest of the presentation:

$$s_i(\mathbf{p}) \equiv \frac{\delta_i \exp\left(p_i\right)^{-\beta}}{\sum_k \delta_k \exp\left(p_k\right)^{-\beta}}$$

Log-logistic demand isn't so different from logistic demand

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$$s_i(\mathbf{p}) \equiv \frac{\delta_i \exp\left(p_i\right)^{-\beta}}{\sum_k \delta_k \exp\left(p_k\right)^{-\beta}}$$

Log-logistic demand is the same, but without the exp,

$$s_i(\mathbf{p}) \equiv \frac{\delta_i p_i^{-\beta}}{\sum_k \delta_k p_k^{-\beta}}$$

Log-logistic demand is H0

If I divide the prices by two, it cancels out

$$s_i(\mathbf{p}/2) \equiv \frac{\delta_i (p_i/2)^{-\beta}}{\sum_k \delta_k (p_k/2)^{-\beta}} = \frac{2^{\beta} \delta_i (p_i)^{-\beta}}{2^{\beta} \sum_k \delta_k (p_k)^{-\beta}} = \frac{\delta_i p_i^{-\beta}}{\sum_k \delta_k p_k^{-\beta}} = s_i(\mathbf{p})$$

Log-logistic demand is homogeneous of degree zero (H0).

Will see it is unique share function with H0, IIA, and monotonicity.

Concrete example with price adjustment

Homogeneity of degree zero equivalent for prices and costs

Firm	Demand	MC	Price	Share
1	$\frac{p_1^{-4}}{p_1^{-4}+16p_2^{-4}}$	1	2	50%
2	$\frac{16p_2^{-4}}{p_1^{-4}+16p_2^{-4}}$	2	4	50%

Cutting everyone's costs by the same amount has no effect on shares

Firm	Demand	MC	Price	Share
1	$\frac{p_1^{-4}}{p_1^{-4}+16p_2^{-4}}$	0.5	1	50%
2	$\frac{16p_2^{-4}}{p_1^{-4}+16p_2^{-4}}$	1	2	50%

Properties and calibration

Log-logistic trades shift invariance of logit for H0

(A1) **Shares:** $\sum_i s_i(s_i(\mathbf{p})) = 1$, $s_i(\mathbf{p}) \geq 0$ for all i

(A2) **Monotonicity:** $s_i(s_i(\mathbf{p}))$ weakly decreasing in p_i and weakly increasing in p_{-i}

(A3) **IIA:** Share in smaller market with subset of firms, M, is independent of prices for firms not in M and $s_i^M(\mathbf{p}^\mathbf{M}) = s_i(s_i(\mathbf{p}))/\sum_{k \in M} s_k(s_i(\mathbf{p}))$

(A4) **Shift invariance:** $s_i(p + \lambda) = s_i(p)$ for $\lambda \geq 0$

(A5) **Homogeneity degree zero:** $s_i(\lambda p) = s_i(p)$ for $\lambda > 0$

Logit equivalent to Axioms 1-4

Log-logit equivalent to Axioms 1-3 and 5

Log-logit is just as easy to calibrate, if not easier!

Logit, $\frac{\delta_i \exp(p_i)^{-\beta}}{\sum_k \delta_k \exp(p_k)^{-\beta}}$, is calibrated with margins, shares, prices like

$$\beta = \frac{1}{M_i p_i (1 - s_i)} \qquad \delta_i = s_i \exp(p_i)^{\beta}$$

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Log-logit, $s_i(\mathbf{p}) \equiv \frac{\delta_i p_i^{-\beta}}{\sum_k \delta_k p_k^{-\beta}}$, is calibrated similarly

$$\beta = \frac{1}{M_i (1 - s_i)} \qquad \delta_i = s_i p_i^{\beta}$$

Conclusion

Log-logit is useful in settings where...

Market-wide changes in costs are central to case

Prices are abstract (e.g., ad load) and changes in value are central to the case

Firms expect large changes in input costs across the industry

Thank You!